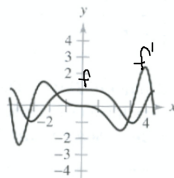


DO NOW

pg 163; #142

142.



3.4 The Chain Rule - Day 6

Derivative Involving Absolute Value:

If u is a differentiable function of x such that $u \neq 0$,

$$\text{then } \frac{d}{dx} [\ln|u|] = \frac{u'}{u}$$

Proof: If $u > 0$, then

$$\frac{d}{dx} [\ln|u|] = \frac{d}{dx} [\ln u] = \frac{u'}{u}$$

If $u < 0$, then

$$\frac{d}{dx} [\ln|u|] = \frac{d}{dx} [\ln(-u)] = \frac{-u'}{-u} = \frac{u'}{u}$$

Example:

Find the derivative of $y = \ln |\csc x|$

$$y' = \frac{-\csc x \cot x}{\csc x}$$

$$y' = -\cot x$$

Log Basics:

$$\text{Recall: } \log_e x = \ln x$$

$$\log_{10} x = \log x$$

How can we rewrite $\log_a x$?

$$\text{Let } \log_a x = n$$

$$a^n = x$$

$$\ln a^n = \ln x$$

$$n \ln a = \ln x$$

$$n = \frac{\ln x}{\ln a}$$

$$\therefore \log_a x = \frac{\ln x}{\ln a}$$

Find the derivative:

$$1. y = \log_7 x$$

$$y = \frac{\ln x}{\ln 7}$$

$$y = \frac{1}{\ln 7} \cdot \ln x$$

$$y' = \frac{1}{\ln 7} \cdot \frac{1}{x}$$

$$y' = \frac{1}{x \ln 7}$$

Find the derivative of

$$2. y = 7^x$$

$$\ln y = \ln 7^x$$

$$\ln y = x \ln 7$$

$$\log_e y = x \ln 7$$

$$y = e^{x \ln 7}$$

$$y' = e^{x \ln 7} \cdot \ln 7$$

$$\boxed{y' = 7^x \cdot \ln 7}$$

$$u = x \ln 7$$

$$u' = \ln 7$$

$$e^{x \ln 7} = y = 7^x$$

Derivatives for bases other than e

$$\frac{d}{dx}[a^x] = (\ln a) a^x$$

$$\frac{d}{dx}[a^u] = (\ln a) a^u \cdot u'$$

$$\frac{d}{dx}[\log_a x] = \frac{1}{x \ln a}$$

$$\frac{d}{dx}[\log_a u] = \frac{u'}{u \ln a}$$

Examples: Find the derivatives.

$$3. f(x) = 5^x \\ f'(x) = (\ln 5) \cdot 5^x$$

$$4. f(x) = 3^{2x} \\ f'(x) = (\ln 3) \cdot 3^{2x} \cdot 2 \\ f'(x) = 2(\ln 3)(3^{2x})$$

$$5. g(x) = 7^{-x}$$

$$g'(x) = (\ln 7)(7^{-x})(-1)$$

$$g'(x) = (-\ln 7)(7^{-x})$$

$$6. y = \log_{10} 5x$$

$$y' = \frac{5}{5x \ln 10}$$

$$y' = \frac{1}{x \ln 10}$$

Find the derivative.

$$7. f(x) = \log_3 \frac{x\sqrt{x-1}}{2} \\ f(x) = \log_3 [x\sqrt{x-1}] - \log_3 2 \\ f(x) = \log_3 x + \frac{1}{2} \log_3 (x-1) - \log_3 2 \\ f'(x) = \frac{1}{x \ln 3} + \frac{1}{2} \cdot \frac{1}{(x-1) \ln 3} - 0 \\ f'(x) = \frac{1}{x \ln 3} + \frac{1}{2(x-1) \ln 3} \\ f'(x) = \frac{1}{\ln 3} \left(\frac{1}{x} + \frac{1}{2(x-1)} \right)$$

$$8. y = \log_{10} \frac{x^2-1}{x}$$

$$y = \log_{10}(x^2-1) - \log_{10} x$$

$$y' = \frac{2x}{(x^2-1) \ln 10} - \frac{1}{x \ln 10}$$

$$y' = \frac{1}{\ln 10} \left(\frac{2x}{x^2-1} - \frac{1}{x} \right)$$

HOMEWORK

pg 162 - 163; 95 -99 odd, 123 - 132
133, 135, 137